STOCHASTIC MODELS
AN ALGORITHMIC APPROACH
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SUMMARY

Wiley Series in Probability and Mathematical Statistics
Editors
Stochastic Models:
An Algorithmic Approach
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Stochastic Models: An Algorithmic Approach fulfills the widely perceived need for an introductory text which demonstrates the effective use of simple stochastic models to gain insight into the behaviour of complex stochastic systems.

The author's earlier book, Stochastic Modeling and Analysis: A Computational Approach (1986) has become a leading text in the fields of applied probability and stochastic optimization. While this new book retains the features of providing theory, realistic examples and practically useful algorithms it is written with a wider readership in mind and is more student-oriented.

Covering renewal and regenerative processes, discrete-time and continuous-time Markov chains, Markovian decision processes, inventory and queuing theory the book will enable students to perform algorithmic analysis for specific problems. Chosen to illustrate the basic models and their associated solution methods, the examples are drawn from a variety of applications fields, such as inventory control, reliability, maintenance, insurance and teletraffic. Each chapter concludes with a range of interesting and thought-provoking exercises, some of which require the use of computer software.

The accessible yet rigorous exposition ensures that the book will be an invaluable resource for senior undergraduate and graduate students of operations research, statistics and engineering.

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TOP
Phase-type distribution. Let us consider Markov process $\tilde{\eta}_t$, $t \geq 0$, with a finite state space $\{1, \ldots, M, M + 1\}$. The states $1, \ldots, M$ are transient and state $M + 1$ is an absorbing state. The service time can be interpreted as a time until the Markov process $\tilde{\eta}_t$, $t \geq 0$, reaches the absorbing state $M + 1$ condition on the fact that the initial state of this process is selected among the transient states acc.