Advanced Mathematical Economics  
(Economia Matemática Avançada)  
PhD in Economics  
Universidade de Lisboa, ISEG  

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Goals  
- Study of functional equations in economics, and, in particular, dynamic systems in the continuum. We will study, at an intermediate level, differential equations (ordinary, partial and stochastic), optimisation of functionals, optimal control of differential equations (ordinary, partial and stochastic) and (possibly) dynamic game theory.  
- Application to some economic theory models from growth theory, dynamic general equilibrium, age-structured models, dynamics of distribution, finance, stochastic dynamics, and dynamic games.  
- If we have time, we will also cover some special topics: models with singularities and models with thresholds.  

Approach  
- We will follow a heuristic approach: emphasise the characterisation of the dynamic properties generated by several dynamic systems rather than trying to prove existence and uniqueness of solutions to the generic functional equations. This is not a course on functional analysis.  
- We will supply study material: as a minimum requirement we will be made available notes for every topic. They do not substitute studying from other literature (referenced or not). A problem set for every topic will also be handed.  
- All material will be posted at [https://pmbrito.github.io/cursos/phd/ame/ame_1920.html](https://pmbrito.github.io/cursos/phd/ame/ame_1920.html). After its initial posting the class notes and the problem sets may be changed along the semester. **Warning: please check the date of the document before downloading.**
Assumed background

- Ideally: mathematics and economic theory at the level of the Masters in Economics, Monetary and Financial Economics, Quantitative Finance (ISEG)
- At least: calculus, algebra, optimisation and probability theory, at an intermediate level.

Topics covered

The main topics which will be covered are the following:

- Ordinary differential equations (ODE)
- Optimal control of ordinary differential equations (OC-ODE)
- Partial differential equations (PDE)
- Optimal control of PDE (OC-PDE)
- Stochastic differential equations (SDE)
- Optimal control of SDE (OC-SDE)

Bibliography

General textbooks covering the topics which will be lectured are:

- **ODE:** Guckenheimer and Holmes (1990), Hale and Koçak (1991), Perko (1996)
- **OC-ODE:** Kamien and Schwartz (1991), Grass et al. (2008), Weber (2011)
- **PDE:** Evans (2010), Olver (2014), Salsa (2016)
- **SDE:** Øksendal (2003), Pavliotis (2014)
- **OC-SDE:** Fleming and Rishel (1975)

Textbooks in macroeconomics and growth theory:

- Growth theory: Acemoglu (2009)
- Macroeconomics: Heijdra (2009), Ljungqvist and Sargent (2012)
- Macro-finance: Stokey (2009)
References


Other references will be given along the way and would be cited in the classnotes.
Assessment

The assessment will be made by a final written closed book exam (see: [https://aquila.iseg.utl.pt/aquila/getFile.do?method=getFile&fileId=296795](https://aquila.iseg.utl.pt/aquila/getFile.do?method=getFile&fileId=296795) regulamento da avaliação dos doutoramentos do ISEG). The questions will be taken from, or will be similar, to the ones included in the problem sets.

Sessions

Tentative scheduling of sessions:

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<tr>
<th>session</th>
<th>date</th>
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<tr>
<td>1</td>
<td>18/02/2020</td>
<td>20:00 - 22:00 Presentation. Introduction.</td>
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<td>2</td>
<td>03/03/2020</td>
<td>20:00 - 22:00 ODE: linear</td>
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<td>3</td>
<td>10/03/2020</td>
<td>20:00 - 22:00 ODE: non-linear - normal forms and bifurcations</td>
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<td>4</td>
<td>17/03/2020</td>
<td>20:00 - 22:00 ODE: non-linear - non-smooth and singular</td>
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<td>5</td>
<td>24/03/2020</td>
<td>20:00 - 22:00 ODE: applications</td>
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<td>6</td>
<td>31/03/2020</td>
<td>20:00 - 22:00 OC-ODE: CV, PMP, DP</td>
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<td>7</td>
<td>14/04/2020</td>
<td>20:00 - 22:00 OC-ODE: extensions and applications</td>
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<td>8</td>
<td>21/04/2020</td>
<td>20:00 - 22:00 PDE: first-order</td>
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<td>9</td>
<td>28/04/2020</td>
<td>20:00 - 22:00 OC-PDE: first-order</td>
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<td>10</td>
<td>05/05/2020</td>
<td>20:00 - 22:00 PDE: parabolic, OC-PDE: parabolic</td>
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<td>12/05/2020</td>
<td>20:00 - 22:00 SDE: linear diffusion equations. OC-SDE</td>
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<td>12</td>
<td>19/05/2020</td>
<td>20:00 - 22:00 PDE and SDE: applications</td>
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Software

Although this is not a course in numerical methods, the use of computers helps a lot in illustrating the solutions, solving, studying the dynamic properties, and estimating the models.

Useful software for solving differential equations:

- public license:
Verified Purchase. Mathematical economics has been around for about 175 years, although as a discipline it has only been recognized for about five decades. Professional economists have had various levels of confidence in its validity and applicability, and mathematical economists have been criticized for the esoteric nature of the mathematics they deploy and some have been ostracized from academic departments for this very reason. This an excellent book primarily for students in Economics either PhD or advanced Masters and/or students interested in Economics and who are coming from “rich in maths” fields. It contains an advanced treatment of the mathematics used in graduate economics. The research undertaken in ISEG’s research centers provides important support for students in Ph.D. programs. For the Ph.D. in Mathematics Applied for Economics and Management the host research centers are: CEMAPRE the Centre for Applied Mathematics and Economics. It is a research unit accredited by the Foundation for Science and Technology (FCT) under the Foundation’s Pluri-annual Program for the Funding of Research and Development Units. The Ph.D. program in Mathematics Applied to Economics and Management is organized into 6 semesters, corresponding to 180 ECTS (credits). In each semester the doctoral student must complete 30 ECTS, which corresponds to a workload of 800 hours. ISEG - Lisbon School of Economics and Management, Universidade de Lisboa. Rua do Quelhas, n.º 6 1200-781 Lisboa, Portugal. Choose a Master, an MS or an MBA in Financial Markets. Advanced Master in Financial Markets. Université Libre de Bruxelles - Solvay Brussels School of Economics and Management. France.