Introduction to Cellular Automata

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Preface

... synthetic universes defined by simple rules...

Tommaso Toffoli & Norman Margolus – Cellular Automata Machines

The history of cellular automata is only quite recent, coming to life at the hands of two fathers, John von Neumann and Stanislaw Ulam in the early 1950s, although it was re-invented several more times, as for example in the work of Konrad Zuse. Subsequent work in the early 1960s included that of Ulam and his coworkers at Los Alamos and by John Holland at the University of Michigan whose work on adaptation continued for several decades. Early theoretical research was conducted by Hedlund (another example of re-invention), Moore, and Myhill, among many others, not always under the name of cellular automata, since the concept was still in its formative stages. A big boost to the popularization of the subject came from John Conway’s highly addictive Game of Life presented in Martin Gardner’s October 1970 column in Scientific American. Still the study of cellular automata lacked much depth, analysis, and applicability and could not really be called a scientific discipline.

All that changed in the early 1980s when physicist Stephen Wolfram in a seminal paper, “Statistical mechanics of cellular automata”, initiated the first serious study of cellular automata. In this work and in a series of subsequent ones Wolfram began producing some of the images that have now become iconic in the field. Conferences were organized and people from various disciplines were being drawn into the field. It is now very much an established scientific discipline with applications found in a great many areas of science. Wolfram has counted more than 10,000 papers referencing his original works on the subject and the field of cellular automata has taken on a life of its own.

The cellular automaton paradigm is very appealing and its inherent simplicity belies its potential complexity. Simple local rules govern an array of cells that update the state they are in at each tick of a clock. It has been found that this is an excellent way to analyze a great many natural phenomena, the reason being that most physical processes are themselves local in
nature — molecules interact locally with their neighbors, bacteria with their neighbors, ants with theirs and people likewise. Although natural phenomena are also continuous, examining the system at discrete time steps does not really diminish the power of the analysis. So in the artificial cellular automaton world we have an unfolding microcosm of the real world.

One of the things self-evident to everyone is the order that is found in Nature. From an ameoba to plants to animals to the universe itself, we find incredible order everywhere. This begs the obvious questions: Where did this order come from — how could it have originated? One of the fundamental lessons of cellular automata is that they are capable of self-organization. From simple local rules that say nothing whatsoever about global behavior, we find that global order is nonetheless preordained and manifest in so many of the systems that we will consider. In the words of theoretical biologist, Stuart Kauffman, it is, “order for free”. It is this order for free that allows us to emulate the order we find in Nature.

Related to the creation of order is the notion of complexity. How can a finite collection of chemicals make up a sentient human being? Clearly the whole is greater than the sum of its parts. How can termites build complex structures when no individual termite who starts a nest even lives to see its completion? The whole field of complexity has exploded over recent years and here too cellular automata play their part. One of the most endearing creatures that we shall encounter is Langton’s Ant in Chapter 6, and this little creature will teach us a lot about complexity.

Of course it is no longer possible in a single text to cover every aspect of the subject. The field, as Wolfram’s manuscript count shows, has simply grown too large. So this monograph is merely an introduction into the brave new world of cellular automata, hitting the highlights as the author sees them. A more advanced and mathematical account can be found in the excellent book by Ilachinski [2002].

One caveat concerning the applications of cellular automata. We are not making any claims that CA models are necessarily superior to other kinds of models or that they are even justified in every case. We are merely presenting them as one way of looking at the world which in some instances can be beneficial to the understanding of natural phenomena. At the very least, I think you will find them interesting. Even if the entire universe is not one monolithic cellular automaton, as at least one scientist believes, the journey to understanding that point of view is well worth the price of admission.

Finally, I wish to thank Auckland University students Michael Brough, Peter Lane, and Malcolm Walsh who produced many of the figures in the text from their cellular automata models and Samuel Dillon who produced the Rule 30 data encryption figures. Their assistance has been invaluable as their programming skills far exceed that of my own. I also wish to thank my daughter-in-law Yuka Schiff for many of the fine graphics and
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Introduction to the Theory of Cellular Automata and One-Dimensional Traffic Simulation. Richard Cochinos. Abstract. This paper is roughly divided into two parts. After a brief introduction, I will discuss the theory and properties underlying cellular automata. Included in this section will be a definition, a list of the physical properties, rules and rules assigning, and a description of some of the dynamical properties inherent in automata. The second section of this paper will be devoted to discussing cellular automata as a model for traffic flow. It will include a one-dimensional example an Cellular Automata. 1983. Introduction. It appears that the basic laws of physics relevant to everyday phenomena are now known. Yet there are many everyday natural systems whose complex structure and behavior have so far defied even qualitative analysis. Cellular automata are a candidate class of such systems. This article surveys their nature and properties, concentrating on fundamental mathematical features. Cellular automata promise to provide mathematical models for a wide variety of complex phenomena, from turbulence in fluids to patterns in biological growth. The general features of their behavior discussed here should form a basis for future detailed studies of such specific systems. Originally published in Los Alamos Science, volume 9, pages 2-21 (Fall 1983).